Math 144 Discussion
10-27-09

Problem 1 *Write out complete solutions to all of the problems in Test #1.*
Proof. *Any questions from the exam?* ■

Suppose that A is an inductive set.

Problem 2 1. *Suppose that B is an inductive set. Why is $A \cap B \neq \varnothing$?*

2. *Suppose that $\mathcal{I}$ is a family of inductive sets. Show that $\cap\mathcal{I}$ is an inductive set.*

3. *Define $\bar{\omega} = \cap\{B \subset A \mid B\}$. From 2.2 we know that $\bar{\omega}$ is an inductive set. Prove that if B is an inductive set, then $\bar{\omega} \subset B$.*

Proof. *1. Since both A and B are inductive sets, $\varnothing \in A$ and $\varnothing \in B$. Thus $A \cap B = \varnothing$.*

2. Clearly for all $I \in \mathcal{I}$, $\varnothing \in I$, thus $\varnothing \in \cap\mathcal{I}$. Suppose $x \in \cap\mathcal{I}$. This means that $x \in I$ for all $I \in \mathcal{I}$. Thus since each I is inductive, $x^+ \in I$ for all $I \in \mathcal{I}$. Thus $x^+ \in \cap\mathcal{I}$ for all $x \in \cap\mathcal{I}$. $\therefore \cap\mathcal{I}$ is inductive.

3. I have no idea what is this even asking... ■

Problem 3 *Suppose that P is a linearly ordered set and $f : P \to P$ is a strictly increasing function (i.e. if $p < q$ then $f(p) < f(q)$). Prove that f is 1-1.*

Proof. *Suppose that $f(p) = f(q)$ and $p \neq q$.*

WLOG assume that $p < q$.

Since $p < q$ and f is strictly increasing we have that $f(p) < f(q)$ a contradiction.

$\therefore f$ is 1-1. ■

Let D be the collection of ducks which are alive at 9:30 AM Pacific time on October 2, 2009 and define the relation $R \subset D \times D$ to include all ordered pairs of ducks who were ever on the same pond at the same time.

Problem 4 1. *Is R reflexive?*

2. *Is R symmetric or anti-symmetric?*

3. Is R transitive?

Proof. *1. Obviously any duck has been in a pond with itself at the same time(unless you are talking about the duck's other self from a parallel universe or something), so R is reflexive.*

2. If duck 1 has been in the same pond as duck 2 at the same time, then clearly duck 2 has been in the same pond as duck 1 at the same time, so R is symmetric.

3. Say Daffy Duck was in some pond with Donald Duck at the same time, and Donald Duck was in some pond at the same time as Plucky Duck. It is not necessarily true that Daffy Duck and Plucky Duck have ever been on the same pond at the same time. ■

Problem 5 *Suppose that A is an infinite subset of* $\mathbb{Q}$*. Prove that there is a bijection from* $\mathbb{Q}$ *onto A.*

Proof. *As A is infinite, it is at least infinitely countable. Since it is a subset of* $\mathbb{Q}$ *it is countable. Thus since A is countable,* $\exists$ *a bijection, call it* η*, from A to* $\mathbb{N}$*. Since* $\mathbb{Q}$ *is countable,* $\exists$ *a bijection, call it* χ*, from* $\mathbb{Q}$ *to* $\mathbb{N}$*. Thus the map* $\eta^{-1} \circ \chi : \mathbb{Q} \to A$ *is a bijection.* ■